Übungsblatt 5 (1)

1) a) $p = p_0\left(1-\frac{a\Delta h}{T_{ext,0}}\right)^{\frac{Mg}{Ra}}$ $\qquad V_0 T_0^{C} = V_E T_E^{C}$

$= p_{ext}$ $\qquad$ reversibel $\Rightarrow p_{innen} = p_{ext}$

$\Rightarrow T_E = T_0\left(\frac{V_0}{V_E}\right)^{1/C}$ $\qquad V_0 = \frac{n_0 R T_0}{p_0}$

$V_E = \frac{n_0 R T_E}{p_0\left(1-\frac{a\Delta h}{T_{ext,0}}\right)^{\frac{Mg}{Ra}}}$

$$\Rightarrow T_E = T_0\left(\frac{\frac{n_0 R T_0}{p_0}}{\frac{n_0 R T_E}{p_0\left(1-\frac{a\Delta h}{T_{ext,0}}\right)^{\frac{Mg}{Ra}}}}\right)^{1/C} = T_0\left(\frac{T_0\cdot\left(1-\frac{a\Delta h}{T_{ext,0}}\right)^{\frac{Mg}{Ra}}}{T_E}\right)^{1/C}$$

$$\Rightarrow T_E^{(1+\frac{1}{C})} = T_0^{(1+\frac{1}{C})}\left(1-\frac{a\Delta h}{T_{ext,0}}\right)^{\frac{Mg}{Ra\cdot C}}$$

$$\Rightarrow T_E = T_0\left(1-\frac{a\Delta h}{T_{ext,0}}\right)^{\frac{Mg}{Ra\cdot C\cdot(1+\frac{1}{C})}} = T_0\left(1-\frac{a\Delta h}{T_{ext,0}}\right)^{\frac{Mg}{Ra(1+C)}}$$

b) ②

$T_E = 6\,°C \,\hat{=}\, 279{,}15\,K \;;\; T_0 = 298\,K \;;\; T_{ext,0} = 288\,K$

$$C = \frac{C_{V,m}}{R} = \frac{20{,}74\,J\,K^{-1}\,mol^{-1}}{8{,}314\,J\,K^{-1}\,mol^{-1}} = 2{,}495$$

$$\left(\frac{T_E}{T_0}\right)^{\frac{Ra(1+c)}{Mg}} = 1 - \frac{a\Delta h}{T_{ext,0}}$$

$$\Rightarrow \Delta h = \left(1 - \left(\frac{T_E}{T_0}\right)^{\frac{Ra(1+c)}{Mg}}\right)\frac{T_{ext,0}}{a}$$

$$\Delta h = \left(1 - \left(\frac{279{,}15\,K}{298\,K}\right)^{\frac{8{,}314\,J\,K^{-1}\,mol^{-1}\cdot 0{,}006\,K\,m^{-1}\,(1+2{,}495)}{0{,}028\,kg\,mol^{-1}\cdot 9{,}81\,m\,s^{-2}}}\right)\frac{288\,K}{0{,}006\,K\,m^{-1}}$$

$$= 1949{,}86\,m$$

Übungsblatt 5 (3)

1)

$\Delta U = \delta W + q$

$\delta W = mgh = 0{,}2\,kg \cdot 9{,}81\,\frac{m}{s^2} \cdot 1{,}55\,m = 3{,}04\,J$

$q = 5\,J$

$\rightarrow \Delta U = 8{,}04\,J$

2)

$$\Delta H_m = \int_{H_{m0}}^{H_m'} dH_m = \int_{T_0}^{T_E} C_{p,m}\,dT = \int_{T_0}^{T_E} \left(a + b\cdot T + \frac{c}{T^2}\right) dT =$$

$$\left[a\cdot T + \tfrac{1}{2} b T^2 - \frac{c}{T}\right]_{T_0}^{T_E} = a\left(T_E - T_0\right) + \tfrac{1}{2} b\left(T_E^2 - T_0^2\right) - c\left(\frac{1}{T_E} - \frac{1}{T_0}\right)$$

$a = 44{,}22\,J\,K^{-1}\,mol^{-1}$ $\quad T_E = 310{,}15$

$b = 8{,}79 \cdot 10^{-3}\,J\,K^{-2}\,mol^{-1}$ $\quad T_0 = 288{,}15\,K$

$c = -8{,}62 \cdot 10^{5}\,J\,K\,mol^{-1}$

$\Rightarrow \Delta H_m = 818{,}49\,J/mol$

3)

$\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T \qquad \alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p \qquad p = \frac{RT}{V_m - b} - \frac{a}{V_m^2} = \frac{nRT}{V - nb} - \frac{an^2}{V^2}$

$$p - \frac{nRT}{V - nb} + \frac{an^2}{V^2} = 0$$

$$\frac{dV}{dp} = -\frac{F_p}{F_V} = -\frac{1}{+\frac{nRT}{(V-nb)^2} - 2\frac{an^2}{V^3}} \Rightarrow \kappa = \frac{1}{\frac{nRTV}{(V-nb)^2} - \frac{2an^2}{V^2}}$$

$$\frac{\partial V}{\partial T} = -\frac{F_T}{F_V} = -\frac{\frac{nR}{V-bn}}{\frac{nRT}{(V-nb)^2} - \frac{2an^2}{V^3}}$$

$$\Rightarrow \alpha = \frac{-\frac{nR}{V-nb}}{-\frac{nRTV}{(V-nb)^2} - \frac{2an^2}{V^2}}$$

$$\frac{\alpha}{\varkappa} = \frac{+\frac{nR}{V-nb}\Big/\cancel{\left(\frac{nRTV}{(V-nb)^2} - \frac{2an^2}{V^2}\right)}}{1\Big/\cancel{\left(\frac{nRTV}{(V-nb)^2} - \frac{2an^2}{V^2}\right)}} = \frac{nR}{V-nb} \Rightarrow \alpha(V_m - b) = \varkappa \cdot R$$

es gilt: $\left(\frac{\partial V}{\partial p}\right)_T \left(\frac{\partial T}{\partial V}\right)_p \left(\frac{\partial p}{\partial T}\right)_V = -1$

$$\Rightarrow \left(\frac{\partial V}{\partial p}\right)_T \cdot \left(\frac{\partial T}{\partial V}\right)_p = -\frac{1}{\left(\frac{\partial p}{\partial T}\right)_V} = -\left(\frac{\partial T}{\partial p}\right)_V = \frac{\left(\frac{\partial V}{\partial p}\right)_T}{\left(\frac{\partial V}{\partial T}\right)_p}$$

$$\Rightarrow \boxed{-\frac{\left(\frac{\partial V}{\partial p}\right)_T}{\left(\frac{\partial V}{\partial T}\right)_p} = \left(\frac{\partial T}{\partial p}\right)_V = \frac{\varkappa}{\alpha}} \Rightarrow \left(\frac{\partial T}{\partial p}\right)_V = -\frac{F_p}{F_T} = -\frac{1}{-\frac{nR}{V-nb}}$$

$$\Rightarrow \frac{\varkappa}{\alpha} = \frac{(V_m - b)}{R}$$

$$\Rightarrow \varkappa \cdot R = \alpha(V_m - b)$$

ideales Gas: $pV = nRT$

$$\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T = \frac{nRT}{Vp^2} = \frac{nRT}{\frac{nRT}{p}\,p^2} = \frac{1}{p}$$

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p = \cancel{\frac{nR}{V^2} = \frac{nR}{\left(\frac{nRT}{p}\right)^2} = \frac{p^2}{nRT^2}}$$

$$\frac{nR}{p\cdot V} = \frac{nR}{nRT} = \frac{1}{T}$$

4)

$$T_E^{\,c}\cdot V_E = T_A^{\,c}\cdot V_A \qquad c = \frac{C_{v,m}}{R} \qquad C_{v,m} = 20{,}74\ \mathrm{J/Kmol}$$

$$\frac{T_E}{T_A} = \left(\frac{V_A}{V_E}\right)^{1/c} = (5)^{1/c} = 1{,}91$$