Übungblatt 3 PC III

① $\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$

(a)

$$\Delta E \geq \frac{\hbar}{2\Delta t} = \frac{\hbar}{2 \cdot 67 \cdot 10^{-18} s} = 7{,}87 \cdot 10^{-19} J$$

(b) $\Delta E = hc\Delta\bar{\nu} \Rightarrow \Delta\bar{\nu} = \frac{\Delta E}{hc} = \frac{7{,}87 \cdot 10^{-19} J}{6{,}626 \cdot 10^{-34} J \cdot 3 \cdot 10^{8} \frac{m}{s}}$

$\Delta\bar{\nu} = 3{,}96 \cdot 10^{6} m^{-1} = 3{,}96 \cdot 10^{4} cm^{-1}$

$\lambda = 500 nm \Rightarrow \bar{\nu} = 20\,000 \; cm^{-1} \Rightarrow$ geht nicht

Zentral wellenlänge muss < 252 nm sein.

Farbe ist in allen Fällen weiß.

② P_i : Wahrscheinlichkeit für Weg i

$P_1 = P_2 = 0{,}4$; $P_3 = 0{,}2$, da $\sum_i P_i = 1$

(a) $\frac{P_1}{P_3} = \frac{0{,}4}{0{,}2} = 2$

(b) $P_i = |c_i|^2 \Rightarrow |c_i| = \sqrt{P_i}$

$\Rightarrow |c_1| = |c_2| = \sqrt{0{,}4} \approx 0{,}632$

$|c_3| = \sqrt{0{,}2} \approx 0{,}447$

$\frac{|c_1|}{|c_3|} = \frac{\sqrt{0{,}4}}{\sqrt{0{,}2}} = \sqrt{2}$

③ (a) $\sin x$ und $\cos x$ im Intervall $[0,\pi]$

$$\int_0^{\pi} \sin x \cos x \, dx = -\int_1^{-1} \cos x \, d\cos x = \int_{-1}^{1} \cos x \, d\cos x = \Big/ t = \cos x \Big/ =$$

$$= \int_{-1}^{1} t \, dt = \frac{t^2}{2}\Big|_{-1}^{1} = \frac{1}{2} - \frac{1}{2} = 0$$

(b) e^{ikx} u e^{inx} im Intervall $[0,2\pi]$

$$\int_0^{2\pi} e^{-ikx} e^{inx} dx = \int_0^{2\pi} e^{-i(k-n)x} dx = \int_0^{2\pi} e^{i(n-k)x} dx =$$

$$\exists\, n = k: \int_0^{2\pi} dx = 2\pi$$

$$\exists\, n \neq k: \int_0^{2\pi} e^{i(n-k)x} dx = \frac{1}{i(n-k)} e^{i(n-k)x}\Big|_0^{2\pi} = 0$$

Die Funktionen sind orthogonal für $n \neq k$

(c) $\sin kx$ und $\cos nx$

$$\sin a + \sin b = 2 \sin \frac{a+b}{2} \cos \frac{a-b}{2}$$

$$\begin{cases} a+b = kx \\ a-b = nx \end{cases} \Rightarrow \begin{cases} a = \frac{k+n}{2} x \\ b = \frac{k-n}{2} x \end{cases}$$

$$\int_0^{2\pi} \sin kx \cos nx \, dx = \frac{1}{2}\Big[\int_0^{2\pi} \sin \frac{k+n}{2} x \, dx + \int_0^{2\pi} \sin \frac{k-n}{2} x \, dx \Big] = 0$$

$\cos kx$ und $\cos nx$

$$\cos a + \cos b = 2\cos\frac{a+b}{2}\cos\frac{a-b}{2}$$

$$\int_0^{2\pi} \cos kx \cos nx \, dx = \frac{1}{2}\int_0^{2\pi} [\cos(n-k)x + \cos(n+k)x] \, dx = 0$$