Übungsblatt 6

① a) zu zeigen: $\hat{H}\Phi_{00} = E\Phi_{00}$

$$\hat{H} = -\frac{1}{2}\hbar\omega\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + V(x) =$$

$$= -\frac{1}{2}\hbar\omega\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - x^2 - y^2\right)$$

$$\hat{H}\Phi_{00} = -\frac{1}{2}\hbar\omega\left(\frac{\partial^2\Phi_{00}}{\partial x^2} + \frac{\partial^2\Phi_{00}}{\partial y^2} - x^2\Phi_{00} - y^2\Phi_{00}\right)$$

// $\frac{\partial^2}{\partial x^2}\Phi_{00} = c(x^2-1)e^{-\frac{1}{2}(x^2+y^2)} = (x^2-1)\Phi_{00}$

$\frac{\partial^2}{\partial y^2}\Phi_{00} = c(y^2-1)e^{-\frac{1}{2}(x^2+y^2)} = (y^2-1)\Phi_{00}$ //

$$\hat{H}\Phi_{00} = -\frac{1}{2}\hbar\omega\left((x^2-1)\Phi_{00} + (y^2-1)\Phi_{00} - x^2\Phi_{00} - y^2\Phi_{00}\right) =$$

$$= -\frac{1}{2}\hbar\omega(x^2-1+y^2-1-x^2-y^2)\Phi_{00} =$$

$$= -\frac{1}{2}\hbar\omega(-2)\Phi_{00} = \hbar\omega\Phi_{00}$$

b) $\hat{H}\Phi_{00} = E\Phi_{00} = \hbar\omega\Phi_{00} \Rightarrow E = \hbar\omega$

c) 1D: $E = \frac{1}{2}\hbar\omega$
2D: $E = \hbar\omega$
$\Rightarrow$ 3D: $E = \frac{3}{2}\hbar\omega$

d) $E_0 = \frac{1}{2}\hbar\omega = \frac{1}{2}\hbar\sqrt{\frac{k}{\mu}} = 3{,}74\cdot10^{-21}\,J$ mit $k = 150\,\frac{N}{m}$, $\mu = 2{,}988\cdot10^{-26}\,kg$

e) $\mu = m(^{12}C) = 12\cdot1{,}661\cdot10^{-27}\,kg = 1{,}993\cdot10^{-26}\,kg$

$$\Delta E = \frac{3}{2}\hbar\omega - \frac{1}{2}\hbar\omega = \hbar\omega = \hbar\sqrt{\frac{k}{\mu}} = \hbar\sqrt{\frac{530\,\frac{N}{m}}{\mu}} = 1{,}72\cdot10^{-20}\,J$$

(2)

Schrödinger-Gleichung:

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\frac{kx^2}{2}\right]\Psi=E\Psi$$

Lösung: $E_i,\ i=1,2\ldots$

Im homogenen Feld:

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\frac{kx^2}{2}+Fx\right]\Psi=E\Psi$$

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\frac{k}{2}\left(x^2+\frac{2F}{k}x\right)\right]\Psi=E\Psi$$

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\frac{k}{2}\left(x^2+2\frac{F}{k}x+\frac{F^2}{k^2}-\frac{F^2}{k^2}\right)\right]\Psi=E\Psi$$

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\frac{k}{2}\left(x^2+2\frac{F}{k}x+\frac{F^2}{k^2}\right)\right]\Psi=\left(E+\frac{F^2}{2k}\right)\Psi$$

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\frac{k}{2}\left(x+\frac{F}{k}\right)^2\right]\Psi=\left(E+\frac{F^2}{2k}\right)\Psi$$

$$z=x+\frac{F}{k}$$

$$dx^2=dz^2$$

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dz^2}+\frac{k}{2}z^2\right]\Psi=E'\Psi$$

$$E_i'=E_i+\frac{F^2}{2k}$$

③ a)

$$L_x = \frac{\hbar}{i}\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$$

$$L_y = \frac{\hbar}{i}\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)$$

$$L_z = \frac{\hbar}{i}\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$$

b) $$[\hat{\ell}_x, \hat{p}_x] = -\hbar^2\left(\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)\frac{\partial}{\partial x} - \frac{\partial}{\partial x}\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)\right) = 0$$

$$[\hat{\ell}_x, \hat{x}] = \frac{\hbar}{i}\left(\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)x - x\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)\right) = 0$$

$$[\hat{x}, \hat{p}_x] = \frac{\hbar}{i}\left(x\frac{\partial}{\partial x} - \frac{\partial}{\partial x}x\right) = -\frac{\hbar}{i}$$

c) $$[\hat{p}_x, \hat{E}_{kin}(x)] = \frac{\hbar}{i}\frac{d}{dx}\cdot\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - \frac{\hbar^2}{2m}\frac{d^2}{dx^2}\cdot\frac{\hbar}{i}\frac{d}{dx} = 0$$

Ja, Impuls und kin. Energie sind gleichzeitig messbar.